

Factoring POW

Find a general solution for the problem, what is the total T of all the factors (excluding the number itself) of a number N where $N = p^k$ for p some prime number and $k \in I$

P = 2			P = 3			P = 5		
$T = p^k - 1$			$T = (p^k - 1)/2$			$T = (p^k - 1)/4$		
N	K	T	N	K	T	N	K	T
2	1	1	3	1	1	5	1	1
4	2	3	9	2	4	25	2	6
8	3	7	27	3	13	125	3	31
16	4	15	81	4	40	625	4	156
$T = (p^k - 1)/(p - 1)$								

Proof:

$$T = \sum_{r=0}^{k-1} p^r$$

$$T = 1 + p + p^2 + \dots + p^{k-2} + p^{k-1}$$

$$Tp = p + p^2 + \dots + \frac{p^k}{p} + p^k$$

$$-T = -1 - p - p^2 - \dots - \frac{p^k}{p}$$

$$Tp - T = p^k - 1$$

Factor T out

$$T(p - 1) = p^k - 1$$

Divide both sides by (p - 1)

$$T = (p^k - 1)/(p - 1)$$